‘Specialization in ICTs and Special Education: Psychopedagogy of Integration’
Postgraduate Program Studies
DEMOKRITUS UNIVERSITY OF THRACE Department of Hellenic Philology
in collaboration with
NCSR DEMOKRITOS Informatics and Telecommunications Institute

GIFTNESS IN MATHEMATICS: RECENT RESEARCH AND ITS UTILITY IN MATHEMATICS TEACHING

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ATHENS 2018

Abstract

In the present review, we attempted to approach the concept of giftness in the field of mathematics education. The basic domain-general and domain-specific theories targeting identification of mathematics giftness are elaborated. The construction of instruments assessing mathematics gifteness is proved to be a difficult process and it is limited to measures of intelligence or/and mathematical ability which can be quantified but, in this way, important aspects of giftness such as creativity are neglected. Moreover, academic performance in mathematics does not necessarily related to mathematics giftness. A possible explanation is that, traditionally, instruction emphasizes and assesses procedural knowledge, namely the knowledge of algorithms and procedures against conceptual knowledge referring to the knowledge of the concepts and principles that govern a domain. Finally, we discuss the need of providing support to mathematics gifted students and redefining the aims of mathematics instruction to encourage creativity in mathematics and facilitate the identification of the gifted in mathematics.

Keywords: Giftness, Mathematics Education, Mathematical ability, Creativity, Conceptual knowledge

References

Ackerman, C. (1997). Identifying gifted adolescents using personality characteristics: Dabrowski's overexcitabilities, *Roeper Review, 19*(4), 229- 236.

Bempeni, M. & Vamvakoussi, X. (2015). Individual differences in students' knowing and learning about fractions: *Evidence from an in-depth qualitative study. Frontline Learning Research, 3,* 17-34.

Bempeni M., Kaldrimidou M., & Vamvakoussi X. (2016). Features of the deep approach to mathematics learning: evidence from exceptional students. In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 2, pp. 75–82. Szeged, Hungary: PME.

Bicknell, B. (2008). Who are the mathematically gifted? Student, parent, and teacher perspectives. *In Proceedings of ICME11.* TG6: Activities and Programs for Gifted Students.

Bloom B. (1985). *Developing talent in young people.* New York: Balantine.

Carman, C. A. (2013). Comparing apples and oranges: Fifteen years of definitions of giftedness in research. *Journal of Advanced Academics, 24*(1), 52-70. doi: 10.1177/1932202X12472602

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn.* Sausalito, CA: Math Solutions.

Ching, T. P. (1997). An experiment to discover mathematical talent in a primary school in Kampong Air. *International Reviews on Mathematical Education, 29*(3), 94–96.

Colangelo, N., Assouline, S. G., & Gross, M. U. M. (2004). *A nation deceived: How schools hold back America's brightest students.* Iowa City, Iowa: The C. Belin & J. N. Blank International Center for Gifted Education and Talent Development.

Coleman, L. J. (2004). Is consensus on a definition in the field possible, desirable, necessary?, *Roeper Review, 27*(1), 10-11.

Csikszentmihalyi, M. (2000). Implications of a systems perspective for the study of creativity. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 313–338). New York: Cambridge University Press.

Csikszentmihalyi, M. & Robinson, R.E. (1990). *The Art of Seeing: An Interpretation of the Aesthetic Experience,* J. Paul Getty Museum, Los Angeles, CA.

Dai, D. Y. (2010). The nature and nurture of giftedness: A new framework for understanding gifted education. New York: Teachers College Press. Gardner, H. (1983/2003). Frames of mind: The theory of multiple intelligences. New York: Basic Books.

Detterman, D. K., & Daniel, M. H. (1989). Correlations of mental tests with each other and with cognitive variables are highest for low IQ groups. *Intelligence, 13,* 349–359.

Demetriou, A., Christou, C., Spanoudis, G., & Platsidou, M. (2002). The development of mental processing: Efficiency, working memory and thinking. *Monographs of the society for research in child development, 67* (1, Serial No. 268). Ανακτήθηκε από from http://www. wiley.com/bw/journal.asp?ref=0037-976x

Entwistle, N., McCune, V., & Tait, H. (2013). Approaches to learning and studying inventory (ASSIST) (3rd edition). Ανακτήθηκε από https://www.researchgate.net/publication/50390092

Ericsson, K. A. & Faivre, R. I. (1988). What's exceptional about exceptional abilities? In L. Obler & D. Fein (Eds.), *The exceptional brain: Neuropsychology of talent and special abilities* (pp. 436-473). New York: Guilfold Press.

Ericsson, K. A. & Krampe, R., & Tesch-Romer, C. (1993). The role of deliberate practice in the acquisition of expert performance. *Psychological Review, 100,* 363-406.

Ericsson K. A. & Lehman, A. C. (1996). Expert and exceptional performance: Evidence of maximal adaptation to task constraints. *Annual Review of Psychology, 47,* 273-305.

Ernest, P. (1985). Special Educational Needs In Mathematics, *CASTME Journal, 6*(1): 22-28.

Feldhusen, J. F. (1986). A conception of giftedness, in: R. J. Sternberg & J. E. Davidson (Eds), *Conceptions of giftedness* (New York, Cambridge University Press), 112-127.

Feldman, D. H. (1986). Giftedness as a developmentalist sees it, in: R. J. Sternberg & J. E. Davidson (Eds), *Conceptions of giftedness* (New York, Cambridge University Press), 285–305.

Gardner, H. (1983). Frames of mind: The theory of multiple intelligences. New York: Basic Books. (New editions 1993, 2004)

Gavin, M. K., Casa, T. M., Adelson, J. L., & Firmender, J. M. (2013). The impact of advanced geometry and measurement units on the achievement of grade 2 students. *Journal of Research in Mathematics Education, 44*(3), 478–510.

Greenes, C., Teuscher, D., & Regis, T. P. (2010). Preparing teachers for mathematically talented middle school students. In M. Saul, S. Assouline, & L. J. Sheffield (Eds.), *The peak in the middle: Developing mathematically gifted students in the middle grades* (pp. 77–91). Reston, VA: NCTM.

Gruber, H. (1986). The self-construction of the extraordinary. In R. J. Stenberg, &J. E. Davidson (Eds.), *Conceptions of giftedness* (pp. 247-263). Cambrige, UK: Cambrige University Press.

Haylock, D. (1997). Recognizing mathematicalcreativity in schoolchildren. *International Reviews on Mathematical Education, Essence of Mathematics 257 29*(3), 68–74.

Hershkowitz, R., Tabach, M., & Dreyfus, T. (2017). Creative reasoning and shifts of knowledge in the mathematics classroom. *ZDM—The International Journal on Mathematics Education, 49*(1), 25–36.CrossRefGoogle Scholar

Hollenstein, H. (1996), "A composite indicator of a firm's innovativeness – an empirical analysis based on survey data for Swiss manufacturing", *Research Policy, 25*(4), 633-45.

Hollingworth, L. S. (1926). Gifted Children: Their Nature and Nurture. New York, NY: Macmillan.

Holton, D., Cheung, K., Kesianye, S., Falk de Losada, M., Leikin, R., Makrides, G., et al. (2009). Teacher development and mathematical challenge. In E. Barbeau & P. Taylor (Eds.), *Challenging mathematics in and beyond the classroom (The 16th ICMI Study)* (pp. 205–242). New York: Springer.

Hong, E., & Aqui, Y. (2004). Cognitive and motivational characteristics of adolescents gifted in mathematics: Comparisons among students with different types of giftedness. *Gifted Child Quarterly,* 48, 191–201.

Hong, E., & Milgram, R. M. (1996). The structure of giftedness: The domain of literature as an exemplar. *Gifted Child Quarterly, 40,* 31–40.

Hong, E., & O'Neil, H. F., Jr. (2001). Construct validation of a trait self-regulation model. *International Journal of Psychology, 36,* 186–194

Gagné, F. (2004). An imperative, but, alas, improbable consensus*!, Roeper Review, 27*(1), 12-14.

Gagné, F. (2009). Building gifts into talents: Detailed overview of the DMGT 2.0. In B. MacFarlane & T. Stambaugh (Eds.), *Leading change in gifted education: The Festschrift of Dr. Joyce Vantassel-Baska* (pp. 61–80). Waco, TX: Prufrock Press.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM: The International Journal on Mathematics Education, 45*(2), 167-181. doi: 10.1007/s11858-012-0467-1.

Kanevsky, L. S. (1990). Pursuing qualitative differences in the flexible use of a problem solving strategy by young children. Journal for the Education of the Gifted, 13, 115–140

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kim, H., Cho, S., & Ahn, D. (2003). Development of mathematical creative problem solving ability test foridentification of gifted in math. *Gifted Education International, 18,* 184–174.

Kontoyianni, K., Kattou, M., Pitta-Pantazi, D., & Christou, C. (2013). Integrating mathematical abilities and creativity in the assessment of mathematical giftedness. *Psychological Test and Assessment Modeling. 55* (3): 289-315.

Koshy V., Ernest P., & Casey R. (2009). *International Journal of Mathematical Education in Science and Technology, 40*(2), 213 – 228.

Krutetskii, V. A. (1968/1976). The psychology of mathematical abilities in schoolchildren (Translated from Russian by Teller, J. Edited by J. Kilpatrick and Wirszup). Chicago: The University of Chicago Press.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Rotterdam: Sense Publishers.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast, 8,* 1–9.

Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM, 45*(2), 183-197.

Livne, M. L., & Milgram, R. M. (2006). Academic versus creative abilities in mathematics: Two components of the same construct? *Creativity Research Journal, 18,* 199-212.

Lubinski, D., & Benbow, C.P. (2000). States of excellence. *American Psychologist, 55,* 137–150.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30,* 236–262.

Mann, E. L. (2009). The Search for Mathematical Creativity: Identifying Creative Potential in Middle School Students. *Creativity Research Journal, 21*(4), 338–348.

Milgram, R., & Hong, E. (2009). Talent loss in mathematics: Causes and solutions. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 149–161). Rotterdam: Sense Publishers.

Miller, R.(1990). Discovering Mathematical Talent. *ERIC Digest* #E482.

National Council of Teachers of Mathematics. (1980). *An agenda for action.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Priciples and Standards for School Mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2001). Adding it up: *Helping children learn mathematics.* In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Washington, DC: Nat. Acad. Press.

National Council of Teachers of Mathematics. (2016/1018). *Research On and Activities For Mathematically Gifted Students,* In F.M. Singer, Sheffield L. M., Freiman V., & Brandl M. (Eds.), ICME-13 Topical Surveys, DOI 10.1007/978-3-319-39450-3_1

Neisser, U., Boodoo, G., Bouchard, T. J., Boykin, A. W., Brody, N., Cesi, S. J., et al. (1996). Intelligence: Knowns and unknowns. *American Psychologist, 51,* 77–101.

Passow, A.H. (1981). The nature of giftedness. *Gifted Child Quarterly, 25,* 5-10.

Pehkonen, E. (1997). The State-of-Art in Mathematical Creativity, *Zentralblatt fur Didaktik der Mathematik, 29*(3), 63-67.

Perish L. (2014). Curriculum in focus: Research guided practice. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia*, pp. 509–516. Sydney: MERGA.

Piechowski, M. M., & Colangelo, N. (1984). Developmental potential of the gifted, Gifted Child Quarterly, 28(2), 80-88.

Pitta-Pantazi, D., Christou, C., Kontoyianni, K., & Kattou, M. (2011). A model of mathematical giftedness: Integrating natural, creative, and mathematical abilities. Canadian Journal of Science, *Mathematics and Technology Education, 11*(1), 39-54.

Pitta-Pantazi, D., Sophocleous, P., & Christou. C. (2013). Spatial visualizers, object visualizersand verbalizers: their mathematical creative abilities, *ZDM,* 45(2), 199-213.

Polya, G. (1962). *Mathematical discover: On understanding, learning and teaching problem solving* (Vol. 1). Hoboken, NJ: John Wiley & Sons.

Raven, J., Raven, J. C., & Court, J. H. (2000, updated 2004). Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 3: The Standard Progressive Matrices. San Antonio, TX: Harcourt Assessment.

Reis, S. M., Westberg, K. L., Kulikowich, J. M., & Purcell, J. H. (1998). Curriculum compacting and achievement test scores: What does the research say? *Gifted Child Quarterly, 42,* 123–129.

Renzulli, J. S. (1976). The Enrichment Triad Model: A guide for developing defensible programs for the gifted and talented. *Gifted Child Quarterly, 20,* 303-326.

Renzulli, J.S. (1978). Whatmakes giftedness? Reexamining a definition. Phi Delta Kappa, 60, 180–184.

Renzulli, J. S. (1986) The three-ring conception of giftedness: a developmental model for creative productivity, in: R. J. Sternberg & J. E. Davidson (Eds) Conceptions of giftedness (New York, Cambridge University Press), 53–92.

Renzulli, J.S. (2002). Emerging Conceptions of Giftedness: Building a Bridge to the New Century. *Except. A Spec. Educ. J., 10,* 67–75.

Ridge, L., & Renzulli, J. (1981). Teaching Mathematics to the Talented and Gifted. In: V.Glennon (Ed.), *The Mathematical Education of Exceptional Children and Youth, An Interdisciplinary Approach* (pp. 191-266). NCTM.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal* of Educational Psychology, 93, 346-362.

Rott, B. & Schindler, M. (2017). Networking Theories on Giftedness—What We Can Learn from Synthesizing Renzulli's Domain General and Krutetskii's Mathematics Specific Theory, *Educ.* Sci. 2017, 7, 6.doi:10.3390/educsci7010006

Ruthven, K. (1987) Ability Stereotyping in Mathematics, *Educational Studies in Mathematics, 18,* 243-253.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist, 23,* 145–166.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334-370). New York, NY: Macmillian.

Sheffield, L. J. (2003). Extending the challenge in mathematics: *Developing mathematical promise in K—8 pupils.* Thousand Oaks, CA: Corwin Press.

Sheffield, L. J. (2006). Developing mathematical promise and creativity. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education, 10*(1), 1–11.

Sheffield, L. (2009). Developing Mathematical Creativity-Questions may be the answer. In R. Leikin, A. Berman, & B. Koichu (Eds.), Creativity in Mathematics and the Education of Gifted Students (pp. 87-100). Rotterdam: Sence Publishers.

Silverman, L. K. (1997). The construct of asynchronous development. *Peabody Journal of Education, 72*(3&4), 36-58.

Singer, F. M. (2007). Beyond conceptual change: Using representations to integrate domain-specific structural models in learning mathematics. *Mind, Brain, and Education, 1*(2), 84–97.

Singer, F. M., Ellerton, N., & Cai, J. (2013). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics, 83*(1), 1–7.

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. Journal of *Secondary Gifted Education, 17*(1), 20–36.

Sriraman, B., & Haavold, P. (2017). Creativity and giftedness in mathematics education: A pragmatic view. In J. Cai (Ed.), First compendium for research in mathematics education. Reston: National Council of Teachers of Mathematics.

Sriraman, B., Haavold, P., & Lee, K. (2013). Mathematical creativity and giftedness: acommentary on and review of theory, new operational views, and ways forward. *ZDM,* 45(2), 215-225.

Stenberg, R. J. (1991). Giftedness according to the triarchic theory of human intelligence. In N. Colangelo & G. A. Davis (Eds.), *Handbook of gifted education* (pp. 45–54). Needham Heights, MA: Allyn & Bacon.

Sternberg, R. J. (1996). What is mathematical thinking? In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 303–318). Mahwah, NJ: Lawrence Erlbaum.

Sternberg, R. J. (1985). *Beyond IQ: a triarchic theory of human intelligence* (Cambridge, MA, Cambridge University Press).

Sternberg, R. J. (2000). Handbook of Creativity. Cambridge University Press.

Tannenbaum, A. J. (1986) Giftedness: a psychosocial approach, in: R. J. Sternberg & J. E. Davidson (Eds), *Conceptions of giftedness* (New York, Cambridge University Press), 21–52.

Terman, L. M. (1906). Stupidity: Genius and A Study of Some of the Intellectual Processes of Seven and Seven "Stupid" Boys. "Bright"Worcester, MA: Clark University. Ανακτήθηκε από https://archive.org/details/geniusstupiditys00term

Terman L. M.(1925). Vol. 1. Genetic Studies of Genius: *Mental and Physical Traits of a Thousand Gifted Children*. Stanford University, CA: Stanford University Press.

Terman, L. M., Lyman, G., Ordahl, G., Ordahl, L. E., Galbreath, N. & Talbert, W. (1917). The Stanford Revision and Extension of the Binet-Simon Scale for Measuring Intelligence. Baltimore, MD: Warwick & York. Ανακτήθηκε από https://archive.org/details/cu31924030584605

Waynberg, A., & Leikin, R. (2009). Multiple solutions to a problem: A tool for assessment of mathematical thinking in geometry. The paper presented at the Sixth Conference of the European Society for Research in Mathematics Education – CERME-6

Wechsler D. (1999), Wechsler Abbreviated Intelligence Scale: Administration Manual. San Antonio, TX: The Psychological Corporation

Wilkinson, G.S. (1993). *Wide Range Achievement Test 3– Administration manual.* Wilmington, DE: Jastak Associates, Inc.

Winner, E. (2000). The origins and ends of giftedness. *American Psych., 55*(1), 159–169.

Winteridge, D. J., Ed., (1989) *A Handbook for Primary Mathematics Coordinators,* London: Paul Chapman Publishing Ltd.

Whitmore, J. R. (1980). Giftedness, Conflict and Underachievement. Boston, NY: Allyn and Bacon, Inc.

Ziegler, A. (2005). The actiotope model of giftedness. In R. J. Steinberg & J. E. Davidson (Eds.), *Conceptions of giftedness* (2nd ed., pp. 422–443). Cambridge, U.K.: Cambridge Univ. Press.

Zimmerman, B. J. (1998) Academic studying and the development of personal skill: a self-regulatory perspective, *Educational Psychologist, 33,* 73–86.

Παιδαγωγικό Ινστιτούτο. (2004). Οδηγός για μαθητές με ιδιαίτερες νοητικές ικανότητες και ταλέντα. Οδηγίες για τους Εκπαιδευτικούς Πρωτοβάθμιας και Δευτεροβάθμιας Εκπαίδευσης. Ανακτήθηκε από http://www.pi-schools.gr/special_education_new/ftp/aps_depps/harismatika.pdf