

**‘Specialization in ICTs and Special Education: Psychopedagogy of Integration’
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**GIFTNESS IN MATHEMATICS: RECENT RESEARCH AND ITS UTILITY
IN
MATHEMATICS TEACHING**

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**POSTGRADUATE
THESIS**

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Abstract

In the present review, we attempted to approach the concept of giftiness in the field of mathematics education. The basic domain-general and domain-specific theories targeting identification of mathematics giftiness are elaborated. The construction of instruments assessing mathematics giftiness is proved to be a difficult process and it is limited to measures of intelligence or/and mathematical ability which can be quantified but, in this way, important aspects of giftiness such as creativity are neglected. Moreover, academic performance in mathematics does not necessarily related to mathematics giftiness. A possible explanation is that, traditionally, instruction emphasizes and assesses procedural knowledge, namely the knowledge of algorithms and procedures against conceptual knowledge referring to the knowledge of the concepts and principles that govern a domain. Finally, we discuss the need of providing support to mathematics gifted students and redefining the aims of mathematics instruction to encourage creativity in mathematics and facilitate the identification of the gifted in mathematics.

Keywords: Giftiness, Mathematics Education, Mathematical ability, Creativity, Conceptual knowledge

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